The Islamia University Of Bahawalpur,
Department of Computer Science & IT
Bahawalnagar Campus
Course: Numerical Analysis **Program:** BSCS-V (Spring 2020)

Topic: Boundary Value Problem

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Boundary Value problems:-
The problem of finding the solution of a differential equation such that all the associated supplementary conditions relate to two different values of the independent variable is called a two-point boundary value problem or "simply boundary value problems".

Boundary value condition: The associated supplementary boundary conditions are called two-point boundary conditions. or simply boundary conditions (B.V.C)

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Boundary value problem

The problem of finding solution of the differential equation such that all the associated supplementary conditions relate to two different values of the indepen-dent variables is called a two-point boundary value problem (or simply a boundary value problem

⇒ Boundary value Conditions

The conditions ~~imposed~~ involve in boundary value problems are called Boundary value condition or simple two-point boundary conditions.

Example of the Boundary value problems.

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note Exp 2

$$\frac{d^2y}{dx^2} + y = 0 \quad \text{— (1)}$$

$y(0) = 1$; $y(\pi) = 5$

G. Sol. $y = A\sin x + B\cos x$ — (2)

For $y(0) = 1$

i.e $x = 0$; $y = 1$ put into eq (2)

$1 = A\sin 0 + B\cos 0$

$1 = 0 + B$

$\Rightarrow \boxed{B = 1}$

Now $y(\pi) = 5$

i.e $x = \pi$, $y = 5$

put into eq (2)

$5 = A\sin\pi + B\cos\pi$

$5 = A(0) + B(-1)$

$5 = -B$

$\Rightarrow B = -5$

We are unable to determine any definite value of 'A'.

Hence this boundary value problem has no solution.

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It means that all boundary value problems need not have solutions

Exp (Same exp with different B.V.C.)
Exp. 1

$$\frac{d^2y}{dx^2} + y = 0 \quad \text{—} (1)$$

$$y(0) = 1 \; ; \; y(\pi/2) = 2$$

verify that $y = c_1 \cos x$ & $y = c_2 \sin x$ are solutions of eq (1)

Sol.

$$y = c_1 \cos x$$

Differentiate w.r.t. 'x'

$$\frac{dy}{dx} = -c_1 \sin x$$

Again differentiate

$$\frac{d^2y}{dx^2} = -c_1 \cos x$$

$$\frac{d^2y}{dx^2} + c_1 \cos x = 0$$

As $y = c_1 \cos x$

So

$$\frac{d^2y}{dx^2} + y = 0$$

which is given differential Equation

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So it is the solution of given Diff. Eq.

Similary $y = C_2 \sin x$
can be verified that it is also solution of given Diff-Eq

i.e G. Sol. is
$$y = C_1 \cos x + C_2 \sin x \quad -(2)$$

Now Apply B.V.C

$y(0) = 1$

At $x = 0$; $y = 1$ put into eq (2)

$$y = C_1 \cos x + C_2 \sin x$$
$$1 = C_1 \cos 0 + C_2 \sin 0$$
$$1 = C_1 + 0$$
$$\Rightarrow \boxed{C_1 = 1}$$

Now $y(\pi/2) = 2$

i.e At $x = \pi/2$; $y = 2$

$$y = C_1 \cos x + C_2 \sin x$$
$$2 = C_1 \cos \pi/2 + C_2 \sin \pi/2$$
$$2 = 0 + C_2$$
$$\Rightarrow \boxed{C_2 = 2}$$

Hence particula solution of eq (1)
is $y = \cos x + 2 \sin x$

Example of the Initial value problems.

I.V.P (19)

Exp No. 4 $\quad \dfrac{dy}{dx} = \dfrac{3x^2+4x+2}{2(y-1)}$

$\cancel{2(y-1)\,dy = (3x^2+4x+2)\,dx}$

$y(0) = -1$

Solve the I.V.P.

Sol.

$$\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)} \quad \text{—(1)}$$

$$2(y-1)\,dy = (3x^2+4x+2)\,dx$$

Taking integration on b.s.

$$2\int (y-1)\,dy = \int (3x^2+4x+2)\,dx$$

$$2\frac{(y-1)^2}{2} = \frac{3x^3}{3} + \frac{4x^2}{2} + 2x + c$$

$$(y-1)^2 = x^3 + 2x^2 + 2x + c \quad \text{—(2)}$$

As $y(0) = -1$

i.e $x = 0$; $y = -1$

put this I.V.c into eq (2)

$$(-1-1)^2 = (0)^3 + 2(0)^2 + 2(0) + c$$

$$\boxed{4 = c}$$

Now required sol.

$$(y-1)^2 = x^3 + 2x^2 + 2x + 4$$

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Exp 05:

$$(1+2y^2)dy = y\cos x\,dx$$

$$y(0) = 1$$

Sol

$$(1+2y^2)dy = y\cos x\,dx \quad -(1)$$

$$\frac{1+2y}{y}dy = \cos x\,dx$$

Integrating on b.s

$$\int \frac{(1+2y^2)}{y}dy = \int \cos x\,dx$$

$$\int \frac{1}{y}dy + \int \frac{2y^2}{y}dy = \int \cos x\,dx$$

$$\ln y + 2\int y\,dy = \sin x + c$$

$$\ln y + 2\frac{y^2}{2} = \sin x + c$$

$$\ln y + y^2 = \sin x + c \quad -(2)$$

Now use I.V.C.

$$y(0) = 1$$

i.e At $x=0$; $y=1$

$$\ln(1) + (1)^2 = \sin 0 + c$$

$$0 + 1 = 0 + c$$

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$c = 1$

put into eq. (2)

$$\ln y + y^2 = \sin x + 1$$

Which required s.l.

Exp 06

$$\frac{dy}{dx} = \frac{x(x^2+1)}{4y^3}$$

$$y(0) = -\frac{1}{\sqrt{2}}$$

Sol.:

$$\frac{dy}{dx} = \frac{x(x^2+1)}{4y^3}$$

$$4y^3\,dy = x(x^2+1)\,dx$$

Integration on both sides.

$$\int 4y^3\,dy = \int x(x^2+1)\,dx$$

$$4\int y^3\,dy = \int x^3\,dx + \int x\,dx$$

$$4\,\frac{y^4}{4} = \frac{x^4}{4} + \frac{x^2}{2} + c$$

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$$y^4 = \frac{x^4}{4} + \frac{x^2}{2} + c \quad - (2)$$

Now I.V.C

$$y(0) = -\frac{1}{\sqrt{2}}$$

i.e at $\cancel{x = -\frac{1}{\sqrt{2}} \; ; \; y = 0}$

$$x = 0 \; ; \; y = -\frac{1}{\sqrt{2}}$$

$$\left(-\frac{1}{\sqrt{2}}\right)^4 = 0 + 0 + c$$

$$\Rightarrow \frac{1}{4} = c$$

put value of 'c' in eq (2)

$$y^4 = \frac{x^4}{4} + \frac{x^2}{2} + \frac{1}{4}$$

Multiply by '4' on b-s

$$4y^4 = x^4 + 2x^2 + 1$$

$$= (x^2)^2 + 2(x^2)(1) + (1)^2$$

$$4y^4 = (x^2+1)^2$$

Taking square root

$$2y^2 = \pm(x^2+1)$$

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$$y^2 = \pm\frac{x^2+1}{2}$$

Again Taking Square root

$$y = \pm\sqrt{\frac{x^2+1}{2}}$$

Which is required solution.

Exp#07

$$\frac{dy}{dx} = \frac{2x}{y+x^2y} \quad \text{—(1)}$$

$$y(0) = -2$$

Sol.

$$\frac{dy}{dx} = \frac{2x}{y(1+x^2)}$$

$$y\,dy = \frac{2x}{1+x^2}\,dx$$

Integrating on both sides,

$$\int y\,dy = \int \frac{2x\,dx}{1+x^2}$$

$$y^2/2 = \ln(1+x^2) + c$$

$$y^2 = 2\ln(1+x^2) + 2c \quad \text{—(2)}$$

As $y(0) = -2$

i.e at $x = 0$; $y = -2$

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Date

$$(-2)^2 = 2\ln(1) + 2c$$

$$4 = 0 + 2c$$

$$\Rightarrow c = 2$$

put into eq (2)

$$y^2 = 2\ln(1+x^2) + 4$$

$$y = \pm\left(2\ln(1+x^2) + 4\right)^{1/2}$$

Now $y = \sqrt{2\ln(1+x^2)+4}$; $y = -\sqrt{2\ln(1+x^2)+4}$

As $y(0) = -2$

i.e at $x = 0$; $y = -2$

$$-2 = \sqrt{2\ln(1)+4}$$

$$-2 = \sqrt{0+4}$$

$-2 \neq 2$ Not satisfy

Now

$$y = -\sqrt{2\ln(1+x^2)+4}$$

$$-2 = -\sqrt{2\ln(1)+4}$$

$$= -\sqrt{0+4}$$

$-2 = -2$ Satisfied so

required sol. is $y = -\left(2\ln(1+x^2)+4\right)^{1/2}$

Best of Luck